SUB-LAPLACIANS WITH DRIFT ON LIE GROUPS OF POLYNOMIAL VOLUME GROWTH

Georgios K. Alexopoulos

Abstract. We prove a parabolic Harnack inequality for a centered sub-Laplacian L on a connected Lie group G of polynomial volume growth by using ideas from Homogenisation theory and by adapting the method of Krylov and Safonov. We use this inequality to obtain a Taylor formula for the heat functions and thus we also obtain Harnack inequalities for their space and time derivatives. We characterise the harmonic functions which grow polynomially. We obtain Gaussian estimates for the heat kernel of L and estimates of the type Berry-Esseen. Finally, we study the associated Riesz transform operators. If L is not centered, then we can conjugate L by a convenient multiplicative function and obtain another centered sub-Laplacian L_C. Thus our results also extend to non-centered sub-Laplacians .

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