

## General arithmetical dimorphy.

### The rings $\mathbb{N}a$ of naturals.

The phenomenon of arithmetical dimorphy begins with the multizetas, but extends far beyond. The reason it tends to be overlooked is that, in order to ensure dimorphic closure, the gap between successive extensions often has to be large, not to say huge.

- The ring of uncoloured multizetas.
- The ring of coloured multizetas.
- The ring of rational polylogarithms or, more precisely, of polylogarithmic integrals with Gaussian rationals as end-points:

$$\text{Wa}^{\alpha_1, \dots, \alpha_l} := (-1)^{l_0} \int_0^1 \frac{dt_l}{\alpha_l - t_l} \cdots \int_0^{t_3} \frac{dt_2}{\alpha_2 - t_2} \int_0^{t_2} \frac{dt_1}{\alpha_1 - t_1} \quad \begin{cases} \alpha_j \in \mathbb{Q} + i\mathbb{Q} \\ l_0 := \sum_{\alpha_i=0} 1 \end{cases}$$

- The rings  $\mathbb{N}a$  of ‘naturals’. There exist rings  $\mathbb{N}a$  of various sizes, but all have this in common: they proceed from some subring  $\mathbb{M}a \subset \mathbb{T}^{\text{a.s.}}$  of *resurgence monomials*  $Ma^\bullet(z)$  and consist of the corresponding *monics*  $Na^\bullet$ , meaning the scalars that feature in the resurgence equations verified by these monomials. Unlike the Zagier-Kontsevich *periods*, which come helter-skelter, in total disarray, without natural indexation and are *a priori* subject to an illimited number of algebraic relations, the ‘naturals’ come with a *natural* indexation and a dimorphic involution.