High order equivalent equations of lattice Boltzmann scheme

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We have used the dispersion equation of a Lattice Boltzmann scheme [LL00] to derive equivalent equations of such methods for several classical schemes (D1Q3 for thermal problem, D1Q3 for acoustic model, d2Q5 for thermal problem). We have compared the results with the direct Taylor apprach suggested by one of us [Du07]. We observe a complete agreement between the two algorithms. Note that intensive use of formal calculus has been necessary for this study. For example, the equivalent equations of the acoustic D1Q3 model are the following ones at the order five (with $\lambda \equiv \Delta x / \Delta t$, $c_s^2 = \alpha \, \lambda^2$ and $\sigma \equiv 1/s - 1/2$) :

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} - \frac{\lambda^2 \, \Delta t^2}{12} (1-\alpha) \frac{\partial^3 q}{\partial x^3} - \frac{\lambda^4 \, \Delta t^3}{12} \sigma \, \alpha \, (1-\alpha) \frac{\partial^4 \rho}{\partial x^4} - \frac{\lambda^4 \, \Delta t^4}{120} (1-\alpha) \, (20\alpha\sigma^2 - 10\sigma^2 - \alpha - 1) \frac{\partial^5 q}{\partial x^5} \, = \, 0$$

$$\frac{\partial q}{\partial t} + \alpha \, \lambda^2 \frac{\partial \rho}{\partial x} - \sigma \, \lambda^2 \, \Delta t \, (1-\alpha) \frac{\partial^2 q}{\partial x^2} - \frac{\lambda^4 \, \Delta t^2}{6} \alpha \, (1-\alpha) \, (6\sigma^2 - 1) \frac{\partial^3 \rho}{\partial x^3} - \frac{\lambda^4 \, \Delta t^3}{12} \sigma \, (1-\alpha) \, (24\alpha \, \sigma^2 - 12\sigma^2 - 4\alpha + 1) \frac{\partial^4 q}{\partial x^4} - \frac{\lambda^6 \, \Delta t^4}{120} \alpha \, (1-\alpha) \, (360\alpha\sigma^4 - 90\alpha\sigma^2 - 240\sigma^4 + 50\sigma^2 + 4\alpha - 1) \frac{\partial^5 \rho}{\partial x^5} \, = \, 0 \, .$$

[*Du*07] F. Dubois. "Equivalent partial differential equations of a lattice Boltzmann scheme", *Computers and Mathematics with Applications*, ICMMES-2005, to appear, 2007.

[*LL*00] P. Lallemand, L.S. Luo. "Theory of the lattice Boltzmann method: Dispersion, dissipation, isotropy, Galilean invariance, and stability", *Physical Review E*, vol. **61**, n° 6, p. 6546-6562, june 2000.

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