Bounce-Back at First Order *

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In [1], we have proposed a method for the analysis of the "bounce-back" boundary condition in the particular case of the d2q9 scheme [2] for a bottom boundary. The incoming particles f_2, f_5 and f_6 are a simple affine function of the corresponding outgoing particules $f_4^*(x)$, f_7^* and f_8^* after relaxation: $f_2(x,t+\Delta t) = f_4^*(x) + \frac{2}{3\lambda} J_y\big(x, t+\frac{\Delta t}{2}\big)$, $f_5(x,t+\Delta t) = f_7^*(x) + \frac{1}{6\lambda}(J_x+J_y)\big(x-\frac{\Delta x}{2}, t+\frac{\Delta t}{2}\big)$ and $f_6(x,t+\Delta t) = f_8^*(x) + \frac{1}{6\lambda}(-J_x+J_y)\big(x+\frac{\Delta x}{2}, t+\frac{\Delta t}{2}\big)$. The functions J_x and J_y are the given values of the momentum on the boundary.

With the previous bounce-back boundary condition, the three expressions
$bb_{24} \equiv f_2^*(x-(0,\Delta x),t) - f_4^*(x)$, $bb_{57} \equiv f_5^*(x-(\Delta x,\Delta x),t) - f_7^*(x)$ and
$bb_{68} \equiv f_6^*(x-(-\Delta x,\Delta x),t) - f_8^*(x)$ are expanded up to the order zero. If we use the Taylor expansions suggested in [3], we can obtain new relations that are precise up to first order accuracy. If α and β define the equilibrium coefficients for energy and squared-energy, *id est* $e^{\rm eq} \equiv \alpha\,\rho$, $e_2^{\rm eq} \equiv \beta\,\rho$, j_x and j_y the two components of the current momentum, q_x^* and q_y^* the two components of the heat flux after relaxation, we can establish without difficulty the following bounce-back relations at first order:

$$bb_{24} = \frac{2}{3\lambda} J_y - \frac{1}{3}\big(q_y^* + \frac{1}{\lambda} j_y\big) - \frac{\Delta x}{36}\,(4-\alpha-2\,\beta)\,\frac{\partial\rho}{\partial y}$$
$$bb_{57} = \frac{1}{6\lambda}(J_x+J_y) + \frac{1}{6}\big(q_x^* + q_y^* + \frac{1}{\lambda}(j_x+j_y)\big) - \frac{\Delta x}{36}\,(4+2\,\alpha+\beta)\,\Big(\frac{\partial\rho}{\partial x}+\frac{\partial\rho}{\partial y}\Big)$$
$$bb_{68} = -\frac{1}{6\lambda}(J_x-J_y) - \frac{1}{6}\big(q_x^* - q_y^* + \frac{1}{\lambda}(j_x-j_y)\big) + \frac{\Delta x}{36}\,(4+2\,\alpha+\beta)\,\Big(\frac{\partial\rho}{\partial x}-\frac{\partial\rho}{\partial y}\Big)\,.$$

We have implemented the three previous relations (and their corresponding variants for other specific locations of the boundary conditions) for a Poiseuille flow with an imposed pressure field at the input and at the output of the domain. In [4], it has been observed (see also [5] and [6]) that super-convergence can be achieved if specific "quartic" combinations of the relaxation coefficients occur. In the present study, we propose modifications of the previous formulae allowing an accurate simulation of the Poiseuille flow for any combination of the relaxation coefficients.

References

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